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Structural Modeling Issues in Flexible Systems

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This paper has the purpose of bringing to light flexibility modeling aspects in connection with multibody dynamic simulation. Toward this end, an examination is made of fundamental issues that arise when one seeks to formulate equations of motion to be used for the dynamics and control simulation of flexible structures. Comparative analyses between component and system modal discretization techniques are presented in the context of multibody flexible systems.

I. Introduction

THE general class of dynamical systems known as flexible multibody systems are assemblages of rigid and elastic bodies. Generally, the motion of such systems is described by a set of hybrid, nonlinear, nonautonomous, and coupled differential equations. To extract meaningful information from this complex mathematical representation, the partial differential equations are first discretized into a finite set of ordinary differential equations via either the classical assumed modes method or the finite element techniques. These technique allow elastic deformations of the system to be approximated by a sum of products of spatial and temporal functions commonly known as shape functions and generalized coordinates, respectively. This study focuses on the choice of these spatial functions.

A. Flexible Multibody System Model

Consider the general flexible multibody system shown in Fig. 1. The equations of motion can be derived by adopting a consistent kinematical procedure to describe the motion. To this end, let us introduce an inertial set of axes X_i, Y_i, Z_i with the origin at I, a set of body axes X_c, Y_c, Z_c with the origin at C and attached to the central body B_c in the undeformed state, and a set of body axes X_a, Y_a, Z_a with the origin at A and attached to the appendage B_a in the undeformed state. For simplicity, we limit the formulation to the central body B_c and appendage B_a. Extension of the formulation to additional appendages is obvious but irrelevant to this particular study. The position vectors of typical points in bodies B_c and B_a can be written as

$$\bar{r}_c = \bar{R}_c + \bar{\rho}_c + \bar{\delta}_c \tag{1}$$

$$\bar{r}_a = \bar{R}_c + \bar{\rho}_{ca} + \bar{\delta}_{ca} + \bar{C}_a^c(\bar{\rho}_a + \bar{\delta}_a) \tag{2}$$

respectively, where $\bar{R}_c$ is the radius vector from I to C, $\bar{\rho}_c$ is the radius vector from C to an elemental mass in the central body, $\bar{\delta}_c$ is the elastic displacement vector of the same elemental mass relative to coordinate frame F_c, $\bar{\rho}_{ca}$ and $\bar{\delta}_{ca}$ are the same vectors evaluated at A, and C_a^c is a transformation matrix used to express the orientation of appendage B_a relative to the coordinate frame attached at C.

The Lagrangian formulation requires the kinetic energy, which in turn requires the velocity of typical points in the various substructures. To derive expressions for these velocities, we assume that the coordinate system F_c is stationary with respect to the inertial frame F_i. This assumption is valid in the present context since we are primarily concerned with the structural deformation interaction between the various components. Thus, by differentiating Eqs. (1) and (2) with respect to time, the velocity vector for a typical point in the central body B_c and appendage B_a can be written as follows:

$$\dot{\bar{r}}_c = \dot{\bar{\delta}}_c \tag{3}$$

$$\dot{\bar{r}}_a = \dot{\bar{\delta}}_{ca} + C_a^c\dot{\bar{\delta}}_a + \dot{C}_a^c(\bar{\rho}_a + \bar{\delta}_a) \tag{4}$$

II. Modal Discretization Algorithms

We assume in this section that the partial differential equations of motion governing the response of such a distributed parameter system are sufficiently complex that an exact solution does not exist or is not feasible. As a result, we are forced to use approximate equations of motion. By modeling the structural deformation using the Rayleigh–Ritz Method, the partial differential equations of motion are discretized with the assumption that the elastic displacement can be expressed in the form

$$\bar{\delta} = \Phi^T(x)\bar{p}(t) \tag{5}$$

where Φ is a matrix of spatial admissible functions and $\bar{p}$ is a vector of temporal generalized displacements. Two discretization methods are considered here, and these are documented in the literature as 1) the assumed modes method and 2) the finite element method.

A. Assumed Modes Method

In the assumed modes approach, a modal set may be selected in order to model the deformation history of the structure. When discretizing at the component level, the system is assumed to be motionless and the various components are allowed to vibrate independently of each other, with user prescribed boundary conditions.

Most often, these component modes are chosen from the set of low-frequency eigenfunctions which are generated by assuming that the component is free or fixed at the points of interconnection with other components. However, these modes are not always statically complete with respect to the component interface, and in many situations the contribution of the high-frequency modes to the response becomes significant. In view of static incompleteness, the component mode set may need to be enlarged until the solution of the system of equations converges thereby increasing the computation time associated with the simulation. Convergence may also be improved by choosing linear combinations of admissible functions capable of satisfying geometric as well as natural boundary conditions.

The use of a consistent kinematical procedure ensures geometric compatibility at boundary points automatically, which is accomplished by defining various sets of body axes so as to guarantee displacement and slope compatibility at boundary points common to any two substructures.

However, provisions must be made for matching natural boundary conditions, such as the bending moment and shearing force at the boundary points between any two substructures. In this regard, a distinction must be made between the substructure B_c and appendage B_a. Indeed, for substructure B_c it is necessary to make

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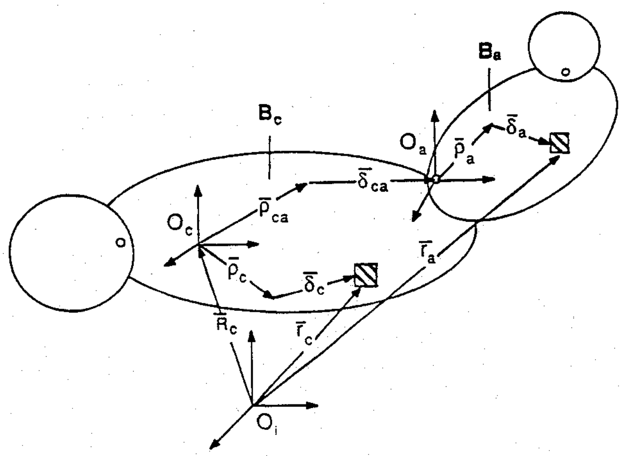


Fig. 1 General flexible multibody system.

provisions for nonzero displacement, slope, bending moment, and shearing force at the boundary point A. On the other hand, because displacement and slope compatibility are guaranteed automatically by the kinematical procedure, for the appendage B_a it is only necessary to make provisions for nonzero bending moment and shearing force at the joint position. All of this can be done by a judicious choice of admissible functions.

Another avenue to represent structural deformation is to discretize using system modes, where the entire structure vibrates in unison such that a motional interaction between the various components of the spacecraft takes place. Intuition suggests that, for a given order of discretization, accuracy of the system dynamics improves as one migrates from the component to system modes. The system representation is physically meaningful, since the modal frequencies represent resonances of the structure; the equations of motion are uncoupled, implying reduced computational effort; and modes can be chosen in a frequency range of interest. The latter technique has not yet been characterized, compared, and documented in the literature in the context of a general flexible multibody dynamics formulation.

The lack of system modes popularity is partly due to the fact that when a multibody system undergoes configurational changes, so do the system modes. Accordingly, many different sets of system modes have to be used, each representing a different instantaneous configuration. This results in enormous amounts of modal data; and performing the transition from one set of data to another in a continuous way is not theoretically available, giving rise to uncertainties and approximations. The large amount of data is unfortunately unavoidable, however, the transition from one set of modal data to the next can be considered acceptable if small maneuver intervals are used and if the maneuver rate is considered small compared to the appendage fundamental frequency, so as to warrant a quasistatic modal solution. This point has been considered by the authors and simulations have been carried out for multibody flexible space structures.[1]

These system modes can be calculated in their entirety, or if the model contains a large number of degrees of freedom so as to make the modeling of the complete system intractable and computationally inefficient, one can resort to the component mode synthesis (CMS), first proposed by Hurty.[2] This technique consists of modeling the motion of the individual substructures. Three types of modes are selected to represent their motion: rigid-body modes, constraint modes, and normal modes. The constraint modes are defined by producing a unit displacement on each redundant constraint in turn, with all other constraints fixed. The normal modes represent the modes of vibration of the substructures with all constraints fixed. Craig and Bampton[3] have subsequently suggested two types of substructure modes, boundary modes providing for displacements and rotations at points along the substructure boundaries, which are related to the constraint modes of Hurty, and modes corresponding to completely restrained boundaries. These substructures modeled individually are made to act together as a single structure by eliminating the redundant generalized coordinates arising from the fact that displacements at points common to two adjacents substructures are included twice in the overall problem formulation, once for each substructure. The elimination process is based on the use of constraint equations resulting from the enforcement of compatibility conditions, which amounts to saying that the displacement of a boundary point shared by two substructures is the same. In yet another approach to the problem, proposed by Benfield and Hruda,[4] the effect of adjacent substructures is accounted for by subjecting a given substructure to inertial and stiffness loadings at the boundaries.

B. Finite Element Method

The discretization of the structure using the hierarchical finite element method is close in nature to the assumed component modes method. In the component modes method, the admissible functions are defined over full substructures, and in the finite element method the local functions are polynomials defined over finite elements. Thus, in the finite element approach, the system is regarded as an assembly of many discrete elements, where each element locally models a portion of the domain. The family of piecewise continuous elements constitutes a finite element model. The nature of the local interpolation functions, the size of the finite elements, and the degree of interelement continuity interact in a relatively complicated way to affect the precision of this approach.

The entire structure is required to move in unison by imposing rotational, translational, and continuity constraints at the joints and boundaries between contiguous elements; in addition, the internal forces are required to balance at the joints. Consequently, the finite element method expresses the displacement of any point in a structure as a function of a finite number of displacements, usually at the boundaries of the elements.

The flexibility of the method lies in the fact that the analyst is free to specify and vary the degree of modeling complexity in the various subsystems that comprise the multibody structure.

The major drawback of the finite element method approach is that many degrees of freedom may be required to obtain a reasonable representation for the structure's deflection. In fact, complex multibody structures will require several thousand degrees of freedom to model it accurately. Fortunately, we are concerned with a few tens of the resulting normal modes of vibration.

For a typical spacecraft modeling problem, the system natural frequencies and corresponding eigenvectors or mode shapes are among the most important final results of a finite element analysis. These, in turn, can be used in the assumed modes method by associating each system mode with a generalized coordinate, as will be demonstrated in the illustrative example to follow.

C. Transformation Matrix C_a^c

The transformation matrix C_a^c, given in Eq. (2), is used to project vectors defined in the local coordinate frame onto the system frame attached to body B_c. This is done to guarantee displacement and slope compatibility between the various substructures, as required by the consistent kinematic procedure adopted in this formulation.

Now, we shall focus on an issue of practical consequence in the derivation and implementation of equations of motion. In the component modes method, C_a^c is a function of the joint flexibility rotation, which translates into a dependence on the generalized coordinates. Thus,

$$C_a^c = C_a^c(\bar{p})$$

which, on substitution into the Lagrangian equations, yields terms like $(\mathrm{d}/\mathrm{d}t)(\partial \dot{C}_a^c/\partial \bar{p})$, $(\partial C_a^c/\partial \bar{p})$, and $(\partial \dot{C}_a^c/\partial \bar{p})$. These terms considerably complicate the formulation process of the equations of motion.

On the other hand, when discretizing using system modes, C_a^c is independent of the generalized coordinates, since the structural deformation is automatically expressed in terms of the coordinate frame attached to the central body B_c. This implies that, when formulating the equations of motion, system modes discretization yields equations of motion in a very simple form and of manageable size, and the equations are uncoupled resulting in a very efficient formulation and computer implementation.

III. Illustrative Example

To illustrate the application of the discretization methods for elastic systems just described, let us consider an L-shaped beam as shown in Fig. 2a. This simple structure has been selected because it is representative of a large class of space and robotic systems comprised of interconnected flexible bodies.

The structural parameters used in this analysis consist of the beam length $L_1 = L_2 = 10$ m, the cross-sectional area $A_1 = A_2 = 3.5$ m^2, the moment of cross-sectional area $I_1 = I_2 = 1$ m^4, and the mass per unit volume $\rho_1 = \rho_2 = 3.75$ kgm^{-3}.

It is proposed to study the dynamical behavior of the structure by assigning three different combinations of bending stiffness properties to the two beams as shown in Table 1.

The dynamic response of beam B_2 at the tip in the x direction was monitored for two different types of forcing functions: the impulse and ramp profile histories; and three different modeling strategies are compared.

1) The system modes method is where each generalized coordinate is associated with a system mode, calculated by the finite element method. The first six system modes are employed in the discretization process for each case study, and these are shown in Fig. 2b. The frequency spectra for the three cases are tabulated in Table 2.

2) The component modes method is where each generalized coordinate is associated with an admissible function in the set used to discretize the body. Three terms in the summation are employed to represent the flexibility of each of the bodies. For body B_1, the first three modes corresponding to a cantilevered beam with an equivalent tip mass are considered. The tip mass is incorporated to simulate the presence of beam B_2 at the end of B_1. For body B_2, the first three modes corresponding to a cantilevered beam are accounted for (Fig. 2c). The frequency spectra for both components are shown in Tables 3 and 4.

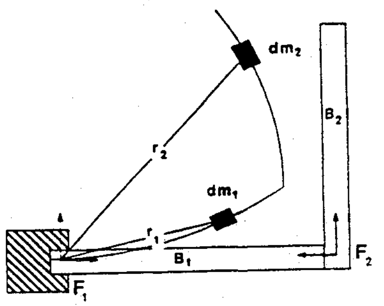
3) The direct integration method is available in the general purpose finite element program ANSYS.[4] In the direct integration, the equations of motion are integrated using a numerical step-by-step procedure, the term direct meaning that prior to the numerical integration, no transformation of the equations into a different form is carried out.[6]

Table 1 Bending stiffness properties for the cantilevered beam

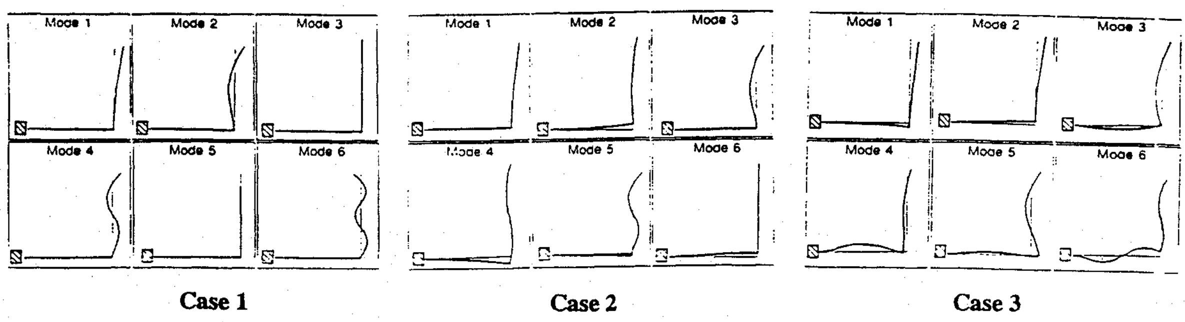
	Case 1	Case 2	Case 3
	E, N·m^{-2}	E, N·m^{-2}	E, N·m^{-2}
B_1	2×10^7	2×10^7	2×10^7
B_2	2×10^3	2×10^5	2×10^7

Table 2 System mode frequencies for the L-shape cantilevered beam

	System mode frequencies, Hz		
	Case 1	Case 2	Case 3
f_1	0.07	0.67	2.31
f_2	0.42	2.82	6.26
f_3	0.57	4.18	28.84
f_4	1.11	10.22	40.42
f_5	1.75	11.20	56.18
f_6	2.07	19.93	93.62



a)



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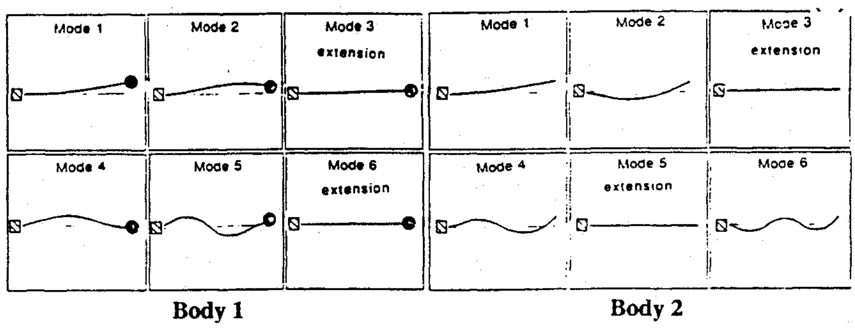

c)

Fig. 2 a) L-shape cantilever beam, b) system modes for the L-shaped beam, and c) component modes used to represent bodies B_1 and B_2.

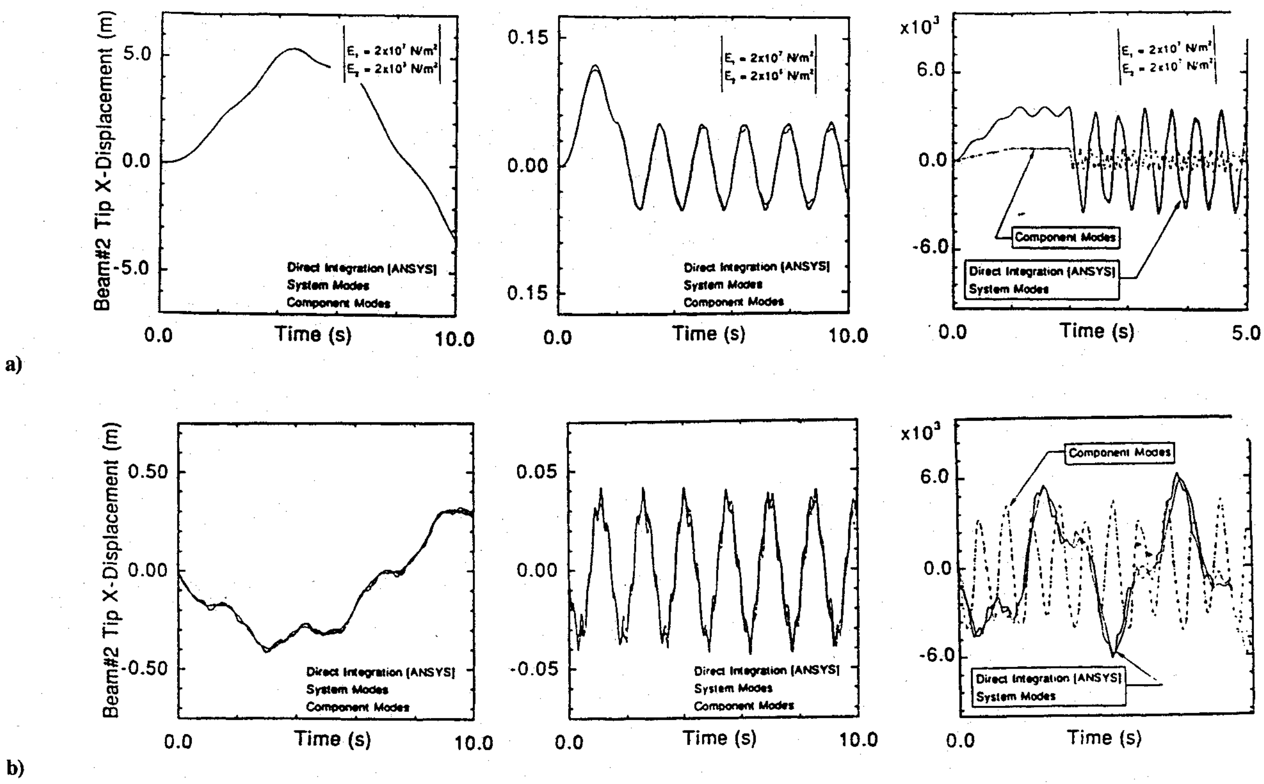

Fig. 3 **Dynamical response of the structure for the three test cases for the three solution procedures in the presence of a) ramp disturbance and b) impulse disturbance.**

Table 3 Component mode frequencies for the beam B_1 modeled as a cantilever with a tip mass

	Component mode frequencies, Hz		
	Case 1	Case 2	Case 3
f_1	2.38	2.38	2.38
f_2	8.30	8.30	8.30
f_3	31.72	31.72	31.72
f_4	49.96	49.96	49.96
f_5	118.63	118.63	118.63
f_6	126.91	126.91	126.91

Table 4 Component mode frequencies for the beam B_2 modeled as a cantilevered beam

	Component mode frequencies, Hz		
	Case 1	Case 2	Case 3
f_1	0.07	0.69	6.92
f_2	0.42	4.20	41.76
f_3	0.58	5.80	58.00
f_4	1.11	11.11	111.19
f_5	1.75	17.53	175.32
f_6	2.03	20.28	202.75

The simulation results, comparing the three methods of solution, are presented in Figs. 3a and 3b. For case study 1, where the frequency spectra of both beams are distinct, there is an excellent correlation between the three methods. The structural response is predominant in the more flexible beam; and the frequency table reveals that the overall system frequencies and the beam B_2 frequencies are comparable. The dynamic simulation shows that the maximum beam transverse tip displacement reaches 0.3 m when the system is subjected to a 5 N · s impulse and that, when a ramp forcing function with a maximum value of 50 N at the end of 2 s is applied, the beam transverse displacement reaches 5 m.

In the second case study, where the frequencies of the individual beams are closer, but still considerably distinct, a slight discrepancy in the results is observed. The finite element direct integration solution and the system modes superposition method agree very well whereas the component modes solution slightly underestimates the amplitude of the response. Nevertheless, the frequency content of all three solutions continue to exhibit an excellent correlation. The table of frequencies for this case shows that there is still a good match between the system frequencies and beam B_2 frequencies. The structural response of the structure when subjected to the impulse and ramp forcing functions already described shows maximum beam transverse displacements of 0.04 and 0.12 m, respectively.

Case study 3 encompasses having both beams of similar structural characteristics. In this case, the component modes method, with three modes used to discretize each beam as in the previous case studies, fails to exhibit a correct dynamic behavior. In fact, whereas the finite element direct integration method and the system modes superposition agree quite well, the component modes solution reveals a reasonable dynamic behavior, but the amplitude and frequency of the solution are completely erroneous. This anomaly was further probed. An attempt was made to improve the component modes solution by increasing the number of modes in the summation, since in theory, by the infinite mode assertion, the solution should converge as the number of terms increases. Six modes were used to represent each beam. Though there was a very small improvement in the results, indicating a trend toward the correct response, the component modes solution still underestimated the magnitude of structural displacement by approximately 50%.

Summarizing, in the component modes solution, a large number of modes is required making the implementation of control algorithms unsuitable, and the boundary conditions have to be judiciously selected for each new set of design parameters to attain an acceptable solution. This particular system would most probably require the beam B_2 to be modeled as a beam with springs at the point of attachment in order to properly account for inertia and stiffness characteristics of beam B_1 at the point of attachment. Thus, in the context of multibody flexible systems, the component modes method is prone to errors if proper steps are not taken to incorporate the appropriate boundary conditions between the various components. This is an obvious disadvantage in the development of automated multibody computer programs and any parametric study would be misleading.

IV. Conclusions

The following conclusions can be drawn from this investigation.

1) In the context of flexible multibody systems, where interconnection between flexible members is an inherent characteristic of the system, it is not possible to generalize on the boundary conditions, thus making this approach very cumbersome and difficult to implement. Moreover, since the transformation matrix C_a^c is dependent on the generalized coordinates, the equations of motions pertaining to flexibility are highly complex and lengthy, resulting in computer simulation effort an order of magnitude higher compared with the other methods.

2) The finite element method, though it offers an accurate and versatile modeling technique for the physical structure, invariably requires a large number of degrees of freedom, making it impractical in the implementation of multibody flexible dynamics and control simulation programs.

3) The system modes technique offers an alternative which takes the best of both methods just described. The generalized coordinates are associated with the system modes of the structure; the system representation is physically meaningful, since the modal frequencies represent resonances of the structure; the flexibility equations of motions are uncoupled, implying reduced computational effort; the formulation is considerably simplified because the transformation matrix C_a^c is not a function of the generalized coordinates; and the effect of boundary conditions between the various substructures is automatically taken into account by the finite element method when calculating the system modes. However, for multibody systems undergoing large configuration changes, a new set of modal data is required for each maneuver step, resulting in enormous amounts of modal data and uncertainties in the transition between modal data intervals.

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